WHITENING OF THE QUARK-GLUON PLASMA

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Parton-parton collisions do not neutralize local color charges in the quark-gluon plasma as they only redistribute the charges among momentum modes. We discuss color diffusion and color conductivity as the processes responsible for the neutralization of the plasma. For this purpose, we first compute the conductivity and diffusion coefficients in the plasma that is significantly colorful. Then, the time evolution of the color density due to the conductivity and diffusion is studied. The conductivity is shown to be much more efficient than the diffusion in neutralizing the plasma at the scale longer than the screening length. Estimates of the characteristic time scales, which are based on close to global equilibrium computations, suggest that first the plasma becomes white and then the momentum degrees of freedom thermalize.

I. INTRODUCTION

Production of the quark-gluon plasma is expected at the early stage of high-energy nucleus-nucleus collision when the energy density is sufficiently high. The experimental data on the so-called elliptic flow [1], which have been obtained at the Relativistic Heavy-Ion Collider (RHIC) in Brookhaven National Laboratory, suggest a surprisingly short, below 1 fm/c [2], equilibration time of the system. Understanding of the thermalization process is thus a key issue of the quark-gluon plasma physics.

In our previous study [3], we have analyzed the local equilibrium of the plasma, which is defined as a state of maximal local entropy. Using the kinetic equations with the collision terms of the Waldmann-Snider form, we have proved that such a state is generically *colorful*, *i.e.* the color four-current is non-vanishing. Thus, the collisions, which are responsible for equilibration of the parton momenta, do *not* neutralize the local color density. Since the color current is (covariantly) conserved in every collision process, the inter-parton collisions redistribute the color charges among various momentum modes but they do not change a local macroscopic color charge. Consequently, if the color charges are not homogeneously distributed in the process of the plasma production due to, say, statistical fluctuations, the inter-parton collisions will not neutralize the system. On the other hand, the global equilibrium of the quark-gluon plasma is locally colorless because of the maximum entropy principle. We assume here that the system does not carry a global color charge and that it does not experience an external chromodynamic field. Once the inter-parton collisions are not responsible for the neutralization of the local charges, one has to invoke other collective mechanism to whiten the plasma. This is the subject of this article.

Local charges are neutralized due to the currents that flow in the system. We consider the diffusive currents generated by the charge density gradient (Fick's law) and the ohmic currents caused by the chromoelectric field (Ohm's law) which is, in turn, induced by the charge density. The color conductivity of the quark-gluon plasma has been studied for long time [4–11], but only recently the problem has been well understood [12–19]. As far as we know, the color diffusion was only briefly discussed in [11]. In all these papers, the plasma near the colorless global

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equilibrium was studied. We are, however, interested in the plasma that is locally colorful. Thus, in Sec. II we derive the diffusion and conductivity coefficients in such a plasma, and then, in Sec. III the temporal evolution of the color charge density is considered. The ohmic currents are shown to be much more efficient than the diffusive ones in neutralizing the local charges.

If not stated otherwise, we follow here the same conventions and notations as in [3].

II. COLOR DIFFUSION AND CONDUCTIVITY COEFFICIENTS

In this section we derive, using transport theory, the diffusive and ohmic currents in a plasma that is locally colorful. The transport equations of quarks, antiquarks and gluons, which form the basis of our analysis, read

$$\big(D^0 + \mathbf{v}\cdot\mathbf{D}\big)Q(\mathbf{p},x) - \frac{g}{2}\{\mathbf{E}+\mathbf{v}\times\mathbf{B}, \nabla_p Q(\mathbf{p},x)\} = C[Q,\bar{Q},G] \;, \tag{1a}$$

$$\big(D^0 + \mathbf{v}\cdot\mathbf{D}\big)\bar{Q}(\mathbf{p},x) + \frac{g}{2}\{\mathbf{E}+\mathbf{v}\times\mathbf{B}, \nabla_p \bar{Q}(\mathbf{p},x)\} = \bar{C}[Q,\bar{Q},G] \;, \tag{1b}$$

$$\big(D^0 + \mathbf{v}\cdot\mathbf{D}\big)G(\mathbf{p},x) - \frac{g}{2}\{\mathbf{E}+\mathbf{v}\times\mathbf{B}, \nabla_p G(\mathbf{p},x)\} = C_g[Q,\bar{Q},G] \;. \tag{1c}$$

The (anti-)quark on-mass-shell distribution functions $Q(\mathbf{p},x)$ and $\bar{Q}(\mathbf{p},x)$, which are $N_c\times N_c$ hermitean matrices, belong to the fundamental representation of the SU(N_c) group, while the gluon distribution function $G(\mathbf{p},x)$, which is a $(N_c^2-1)\times(N_c^2-1)$ matrix, belongs to the adjoint representation. The covariant derivative $D^\mu \equiv \partial^\mu - ig[A^\mu(x),\cdots]$, the chromoelectric and chromomagnetic fields, $\mathbf{E}$ and $\mathbf{B}$, which enter the transport equations also belong to either the fundamental or adjoint representation, correspondingly. To simplify the notation, and differently than in [3], we use the same symbols D^0, $\mathbf{D}$, $\mathbf{E}$ and $\mathbf{B}$ to denote a given quantity in the fundamental or adjoint representation. $x\equiv(t,\mathbf{x})$ denotes the four-position while $\mathbf{p}$ is the three-momentum. Because the partons are assumed to be massless, the velocity $\mathbf{v}$ equals $\mathbf{p}/|\mathbf{p}|$. The collision terms C, $\bar{C}$ and C_g will be discussed later on.

We are interested in a state close to the colorful local equilibrium. When the effects of quantum statistics are neglected, the (on-mass-shell) local equilibrium distribution functions read [3]

$$Q^{\rm eq}(\mathbf{p},x) = \exp\Big[-\beta(x)\big(u_\nu(x)p^\nu - \mu_b(x) - \widetilde{\mu}(x)\big)\Big] \;, \tag{2a}$$

$$\bar{Q}^{\rm eq}(\mathbf{p},x) = \exp\Big[-\beta(x)\big(u_\nu(x)p^\nu + \mu_b(x) + \widetilde{\mu}(x)\big)\Big] \;, \tag{2b}$$

$$G^{\rm eq}(\mathbf{p},x) = \exp\Big[-\beta(x)\big(u_\nu(x)p^\nu - \widetilde{\mu}_g(x)\big)\Big] \;, \tag{2c}$$

where $p^\mu = (E_p,\mathbf{p})$, and $E_p = |\mathbf{p}|$ for massless quarks and antiquarks, and for gluons; $\beta = 1/T$, u^μ, μ_b denote, respectively, the inverse temperature, hydrodynamic velocity and baryon chemical potential; the colored chemical potentials of quarks ($\widetilde{\mu}$) and of gluons ($\widetilde{\mu}_g$) obey the relationship

$$\widetilde{\mu}_g(x) = 2T^a{\rm Tr}[\tau^a\widetilde{\mu}(x)] = \mu_a(x)T^a \;, \tag{3}$$

where τ^a, T^a with $a=1,\dots,N_c^2-1$ are the SU(N_c) group generators in the fundamental and adjoint representations, normalized as ${\rm Tr}[\tau^a\tau^b] = \frac{1}{2}\delta^{ab}$ and ${\rm Tr}[T^aT^b] = N_c\delta^{ab}$.

In the local equilibrium state there is a non-vanishing color charge density, which is defined as

$$\rho(x) = -\frac{g}{2}\int\frac{d^3p}{(2\pi)^3}\Big(q(\mathbf{p},x) - \bar{q}(\mathbf{p},x) + 2\tau^a{\rm Tr}\big[T^aG(\mathbf{p},x)\big]\Big) \;, \tag{4}$$

with

$$q(\mathbf{p},x) \equiv Q(\mathbf{p},x) - \frac{1}{N_c}{\rm Tr}\big[Q(\mathbf{p},x)\big] \;, \qquad \bar{q}(\mathbf{p},x) \equiv Q(\mathbf{p},x) - \frac{1}{N_c}{\rm Tr}\big[\bar{Q}(\mathbf{p},x)\big] \;.$$

In the local rest frame, where $u^\mu = (1,0,0,0)$, the color current defined as

$$\mathbf{j}(x) = -\frac{g}{2}\int\frac{d^3p}{(2\pi)^3}\,\mathbf{v}\Big(q(\mathbf{p},x) - \bar{q}(\mathbf{p},x) + 2\tau^a{\rm Tr}\big[T^aG(\mathbf{p},x)\big]\Big) \;, \tag{5}$$

vanishes because the distribution functions are locally isotropic.

We now study the system for long time scales. We consider small deviations from local equilibrium, and we write down the quark distribution function as $Q(\mathbf{p},x) = Q^{\rm eq}(\mathbf{p},x) + \delta Q(\mathbf{p},x)$. Assuming that

$$|Q^{\rm eq}| \gg |\delta Q| \;, \qquad |D^0 Q^{\rm eq}| \gg |D^0 \delta Q| \;, \qquad |\mathbf{D} Q^{\rm eq}| \gg |\mathbf{D} \delta Q| \;, \qquad |\nabla_p Q^{\rm eq}| \gg |\nabla_p \delta Q| \;, \tag{6}$$

and taking into account the local isotropy of the equilibrium state, the transport equation (1a) can be approximated as

$$\big(D^0 + \mathbf{v}\cdot\mathbf{D}\big)\, Q^{\rm eq} + \frac{g}{2}\{\mathbf{E}, \nabla_p Q^{\rm eq}\} = L[\delta Q] \;, \tag{7}$$

where $L[\delta Q]$ is the collision term linearized around the local equilibrium distribution function. Analogous equations hold for antiquarks and gluons. Let us recall here that the collision terms evaluated for the local equilibrium distribution functions (2) vanish [3].

To get the transport coefficients of color diffusion and conductivity we assume that the linearized collision terms L, $\bar{L}$ and L_g satisfy the relationship

$$\int \frac{d^3p}{(2\pi)^3}\, \mathbf{v}\, L[\delta Q] = -\gamma \int \frac{d^3p}{(2\pi)^3}\, \mathbf{v}\, \delta Q \;, \tag{8}$$

where $1/\gamma$ is the characteristic relaxation time which, for simplicity, is assumed to be the same for quarks, antiquarks and gluons. Our analysis based on the linearized transport equation (7) with Ansatz (8) is valid for $t \gtrsim 1/\gamma$. The relation (8) is obeyed [16] by the Waldmann-Snider collision term linearized around global (colorless) equilibrium. Such a linear collision term turns out to be non-local in velocities, allowing for the covariant color current conservation [12]. A similar linearization around the local colorful equilibrium is, in principle, feasible, but it appears rather difficult as it requires knowledge of the scattering matrix elements computed in the colorful background.

The relationship (8) is trivially satisfied by the collision term in the relaxation time approximation

$$C^{\rm RTA}[Q,\bar{Q},G] = \frac{1}{\tau}\big(Q^{\rm eq}(\mathbf{p},x) - Q(\mathbf{p},x)\big) = -\frac{1}{\tau}\,\delta Q(\mathbf{p},x) = L^{\rm RTA}[\delta Q] \;, \tag{9}$$

with $\tau = 1/\gamma$. Unfortunately, this approximation is known to contradict the covariant current conservation as C, $\bar{C}$ and C_g in the form (9) violate the collisional invariant

$$\int \frac{d^3p}{(2\pi)^3}\, \big(C - \bar{C} + 2\tau^a {\rm Tr}[T^a C_g]\big) = 0 \;. \tag{10}$$

However, the collision term (9) can be improved to a form that complies with the condition (10). In analogy to the so-called BGK collision term [20], we have found the expression

$$\begin{aligned} C^{\rm BGK}[Q,\bar{Q},G] &= \frac{1}{\tau}\Big(Q(\mathbf{p},x) - N(x)\,\big(N^{\rm eq}(x)\big)^{-1} Q^{\rm eq}(\mathbf{p},x)\Big) \\ &\approx -\frac{1}{\tau}\Big(\delta Q(\mathbf{p},x) - \delta N(x)\,\big(N^{\rm eq}(x)\big)^{-1} Q^{\rm eq}(\mathbf{p},x)\Big) = L^{\rm BGK}[\delta Q] \;, \end{aligned} \tag{11}$$

with

$$N(x) \equiv \int \frac{d^3p}{(2\pi)^3}\, Q(\mathbf{p},x) \;, \tag{12}$$

and $(N^{\rm eq})^{-1}$ being the inverse matrix of $N^{\rm eq}$. One easily shows that the collision terms of the form (11) satisfy the constraint (10), as

$$\int \frac{d^3p}{(2\pi)^3}\, C^{\rm BGK} = \int \frac{d^3p}{(2\pi)^3}\, \bar{C}^{\rm BGK} = \int \frac{d^3p}{(2\pi)^3}\, C_g^{\rm BGK} = 0 \;.$$

The collision term (11) also obeys the relation (8). It still violates the energy-momentum conservation law, but it can be further improved [20].

Using the relation (8), the color current generated by deviations from equilibrium is

$$
\begin{aligned}
\mathbf{j}(x) &= -\frac{g}{2}\int \frac{d^3p}{(2\pi)^3}\,\mathbf{v}\Big(\delta q(\mathbf{p},x) - \delta\bar{q}(\mathbf{p},x) + 2\tau^a \mathrm{Tr}\big[T^a \delta G(\mathbf{p},x))\big]\Big) \qquad (13)\\
&= \frac{g}{2\gamma}D^0 \int \frac{d^3p}{(2\pi)^3}\,\mathbf{v}\Big(q^{\rm eq} - \bar{q}^{\rm eq} + 2\tau^a \mathrm{Tr}\big[T^a G^{\rm eq}\big]\Big)\\
&+ \frac{g}{2\gamma}\int \frac{d^3p}{(2\pi)^3}\,\mathbf{v}(\mathbf{v}\cdot\mathbf{D})\Big(q^{\rm eq} - \bar{q}^{\rm eq} + 2\tau^a \mathrm{Tr}\big[T^a G^{\rm eq}\big]\Big)\\
&- \frac{g^2}{4\gamma}\int \frac{d^3p}{(2\pi)^3}\,\mathbf{v}\Big(\{\mathbf{E},\nabla_p(Q^{\rm eq}+\bar{Q}^{\rm eq})\} - \frac{2}{N_c}\mathrm{Tr}\big[\mathbf{E}\cdot\nabla_p(Q^{\rm eq}+\bar{Q}^{\rm eq})\big]\Big)\\
&- \frac{g^2}{2\gamma}\tau^a \int \frac{d^3p}{(2\pi)^3}\,\mathbf{v}\,\mathrm{Tr}[T^a\{\mathbf{E},\nabla_p G^{\rm eq}\}]\,.
\end{aligned}
$$

One observes that the term with D^0 drops out as the color current vanishes in local equilibrium. We also take into account that the local equilibrium is isotropic and we perform partial integrations in the terms with the chromoelectric field. Putting additionally $\mathbf{v}^2 = 1$, the current gets the form

$$
\begin{aligned}
\mathbf{j} &= \frac{g}{6\gamma}\,\mathbf{D}\int \frac{d^3p}{(2\pi)^3}\Big(q^{\rm eq} - \bar{q}^{\rm eq} + 2\tau^a \mathrm{Tr}\big[T^a G^{\rm eq}\big]\Big) \qquad (14)\\
&+ \frac{g^2}{6\gamma}\int \frac{d^3p}{(2\pi)^3}\,\frac{1}{E_p}\Big(\{\mathbf{E},(Q^{\rm eq}+\bar{Q}^{\rm eq})\} - \frac{2}{N_c}\mathrm{Tr}[\mathbf{E}\,(Q^{\rm eq}+\bar{Q}^{\rm eq})]\Big)\\
&+ \frac{g^2}{3\gamma}\,\tau^a \int \frac{d^3p}{(2\pi)^3}\,\frac{1}{E_p}\mathrm{Tr}[T^a\{\mathbf{E},G^{\rm eq}\}]\,,
\end{aligned}
$$

which can be written as

$$
\mathbf{j} = -\mathcal{D}\mathbf{D}\rho + \frac{1}{2}\big(\{\sigma_q,\mathbf{E}\} - \frac{2}{N_c}\mathrm{Tr}[\sigma_q\,\mathbf{E}]\big) + \tau^a \mathrm{Tr}[T^a\{\sigma_g,\mathbf{E}\}]\,, \qquad (15)
$$

where the diffusion constant $\mathcal{D}$ and conductivity coefficients σ_q, σ_g are

$$
\mathcal{D} = \frac{1}{3\gamma}\,, \qquad \sigma_q = \frac{g^2}{3\gamma}\int \frac{d^3p}{(2\pi)^3}\,\frac{1}{E_p}\,\Big(Q^{\rm eq}+\bar{Q}^{\rm eq}\Big)\,, \qquad \sigma_g = \frac{g^2}{3\gamma}\int \frac{d^3p}{(2\pi)^3}\,\frac{1}{E_p}\,G^{\rm eq}\,. \qquad (16)
$$

As seen, the current (15) is traceless as it should. It gets a much simpler form in the adjoint representation. Namely, for $\mathbf{j}^a = 2\mathrm{Tr}[\tau^a \mathbf{j}]$, we get

$$
\mathbf{j}^a(x) = -\mathcal{D}\,\mathbf{D}^{ab}\rho^b(x) + \sigma^{ab}\mathbf{E}^b(x)\,, \qquad (17)
$$

where $\mathbf{D}^{ab} = \delta^{ab}\nabla + g f^{acb}\mathbf{A}^c$, and the color conductivity tensor reads

$$
\sigma^{ab} = \frac{g^2}{3\gamma}\int \frac{d^3p}{(2\pi)^3}\,\frac{1}{E_p}\,\Big(\mathrm{Tr}[\{\tau^a,\tau^b\}(Q^{\rm eq}+\bar{Q}^{\rm eq})] + \mathrm{Tr}[\{T^a,T^b\}G^{\rm eq}]\Big)\,. \qquad (18)
$$

The diffusion constant is, as previously, $1/3\gamma$. When the equilibrium is colorless, the conductivity σ is proportional to the unit matrix in color space, but for a colorful configuration it is not.

It is interesting to note that the chromoelectric field contributes to the induced baryon current. Repeating the analysis fully analogous to that of the color current, one finds that the baryon current defined as

$$
\mathbf{b}(x) = \frac{1}{3}\int \frac{d^3p}{(2\pi)^3}\,\mathbf{v}\,\mathrm{Tr}[Q(\mathbf{p},x) - \bar{Q}(\mathbf{p},x)]\,. \qquad (19)
$$

equals

$$
\mathbf{b}(x) = -\mathcal{D}\nabla b(x) - \frac{2}{3g}\mathrm{Tr}[\sigma_q\,\mathbf{E}(x)]\,, \qquad (20)
$$

where b is the baryon density while $\mathcal{D}$ and σ_q are given by Eqs. (16). When the equilibrium is colorless, the conductivity σ is proportional to the unit matrix in color space, and the effect of the chromoelectric field on the baryon current disappears.

III. TEMPORAL EVOLUTION OF THE COLOR DENSITY

Our aim here is to discuss how a locally colorful quark-gluon plasma becomes white. As an introduction to our chromodynamic considerations, we first discuss the temporal evolution of the electric charge density in the electromagnetic plasma.

A. Diffusion vs. conductivity - electrodynamic case

The electric current ($\mathbf{j}$) generated by both the gradient of charge density (ρ) and the electric field ($\mathbf{E}$) is

$$\mathbf{j}(x) = -\mathcal{D}\,\nabla\rho(x) + \sigma\,\mathbf{E}(x)\ . \tag{21}$$

$\mathcal{D}$ and σ are assumed here to be the transport coefficients derived in a semi-static limit as in Sec. II. Therefore, Eq. (21) holds for slowly varying $\rho(x)$ and $\mathbf{E}(x)$.

Taking into account the Gauss law $\nabla\mathbf{E}(x) = \rho(x)$, the current conservation $\partial\rho/\partial t + \nabla\mathbf{j} = 0$ combined with Eq. (21) provides the equation

$$\Big[\frac{\partial}{\partial t} - \mathcal{D}\,\nabla^2 + \sigma\Big]\rho(x) = 0\ , \tag{22}$$

which, supplemented by the initial condition $\rho(0,\mathbf{x}) = \rho_0(\mathbf{x})$, is solved by

$$\rho(x) = e^{-\sigma t}\,n(x) \tag{23}$$

with n satisfying the diffusion equation

$$\Big[\frac{\partial}{\partial t} - \mathcal{D}\,\nabla^2\Big]n(x) = 0\ , \tag{24}$$

and the initial condition $n(0,\mathbf{x}) = \rho_0(\mathbf{x})$. Eq. (22) can be easily solved by means of the Fourier transformation as

$$\rho(x) = \int \frac{d^3k}{(2\pi)^3}\, e^{-(\sigma+\mathcal{D}\mathbf{k}^2)t + i\mathbf{k}\mathbf{x}}\rho_0(\mathbf{k})\ , \tag{25}$$

where $\rho_0(\mathbf{k})$ is the Fourier transform of the initial charge density

$$\rho_0(\mathbf{k}) = \int d^3x\, e^{-i\mathbf{k}\mathbf{x}}\rho_0(\mathbf{x})\ . \tag{26}$$

It is assumed here that the integral (26) exsists which requires vanishing of $\rho_0(\mathbf{x})$ when $|\mathbf{x}| \to \infty$.

We note that the solution (25) can be also written down as

$$\rho(x) = \frac{1}{(4\pi\mathcal{D}t)^{3/2}} \int d^3x'\, \exp\Big(-\sigma t - \frac{(\mathbf{x}-\mathbf{x}')^2}{4\mathcal{D}t}\Big)\rho_0(\mathbf{x}')\ , \tag{27}$$

where the Green's function

$$G(\mathbf{x},\mathbf{x}',t) = \frac{1}{(4\pi\mathcal{D}t)^{3/2}}\exp\Big(-\sigma t - \frac{(\mathbf{x}-\mathbf{x}')^2}{4\mathcal{D}t}\Big)\ , \tag{28}$$

represents the charge density which obeys Eq. (22) and equals $\delta^{(3)}(\mathbf{x}-\mathbf{x}')$ at $t=0$.

The solution (25) shows that the charge density modes of all $\mathbf{k}$ decay exponentially. The long-wavelength modes with $\mathbf{k}^2 < \sigma/\mathcal{D}$ are dominantly neutralized by the ohmic currents while those with $\mathbf{k}^2 > \sigma/\mathcal{D}$ are neutralized due to the diffusion. It should be remembered, however, we can trust the solution (25) or (27) only for sufficiently long time intervals because Eq. (21) holds for slowly varying $\rho(x)$ and $\mathbf{E}(x)$. We return to this point in the next section where it is discussed quantitatively in the context of quark-gluon plasma.

B. Diffusion vs. conductivity - chromodynamic case

The color current generated by both the gradient of color density and the chromoelectric field is given by Eq. (17). The covariant current conservation $D^0\rho + \mathbf{Dj} = 0$ combined with the Gauss law $\mathbf{DE}(x) = \rho(x)$, provides the equation

$$\big[D^0 - \mathcal{D}\,\mathbf{D}^2 + \sigma\big]\rho(x) = 0 \;, \tag{29}$$

where the term $[\mathbf{D}, \sigma]\,\mathbf{E}$ has been neglected. In the Appendix we show that $[\mathbf{D}, \sigma]\,\mathbf{E} \ll \sigma\mathbf{DE}$ in the small coupling limit. All chromodynamic quantities discussed in this section belong to the adjoint representation, and thus, the color indices are suppressed.

Eq. (29) can be treated as its Abelian counterpart (29) if $[D^0, \sigma] = 0$. In the Appendix this commutator is shown to be indeed small. Thus, Eq. (29) is solved by

$$\rho(x) = e^{-\sigma t}\, n(x) \tag{30}$$

with n satisfying the diffusion equation

$$\big[D^0 - \mathcal{D}\,\mathbf{D}^2\big]n(x) = 0 \;. \tag{31}$$

$\rho(x)$ and $n(x)$ obey the initial condition $\rho(0, \mathbf{x}) = n(0, \mathbf{x}) = \rho_0(\mathbf{x})$. Thus, we expect that, as in the electromagnetic case, the charge density decays exponentially and the conductivity dominates over the diffusion for the modes with $\mathbf{k}^2 < \sigma/\mathcal{D}$.

For further discussion one needs an estimate of the relaxation time $1/\gamma$ which controls both σ and $\mathcal{D}$. However, the reliable estimate can be given only for the quark-gluon plasma close to global (colorless) equilibrium of very high temperatures where $1/g \gg 1$. Then, the color conductivity is of order [9,16]

$$\sigma \;\sim\; \frac{T}{\ln(1/g)} \;. \tag{32}$$

According to Eq. (18), the conductivity, due to the dimensional argument, can be approximated as $\sigma \sim g^2T^2/\gamma$ which combined with the estimate (32) provides

$$\frac{1}{\gamma} \sim \frac{1}{g^2 \ln(1/g)\, T} \sim t_{\rm soft} \;, \tag{33}$$

where $t_{\rm soft}$ is the characteristic time scale of the of parton-parton collisions at momentum transfers of order g^2T [16]. We also observe that

$$\frac{\sigma}{\mathcal{D}} \sim g^2T^2 \sim m_D^2 \;, \tag{34}$$

where m_D is the screening mass. Having these estimates, we first note that all modes of charge density longer than the screening length are neutralized dominantly by the ohmic currents. However, we can trust Eqs. (30,31) only for time intervals longer than $1/\gamma$ because the derivation of the color conductivity presented in Sec. II is valid for $t \gtrsim 1/\gamma$. Taking into account the estimate (32), we find that all modes of charge density vanish at $t \gtrsim 1/\gamma$. Since the characteristic time scale of color dissipation cannot be shorter than $t_{\rm soft}$, the whitening of the quark-gluon plasma occurs at $t \sim t_{\rm soft}$. At shorter times scales the color density is expected to oscillate. We note that in the electromagnetic plasma local charges are neutralized very fast, but the currents survive in the system for a long time as the plasma is a very good conductor. Although, the quark-gluon plasma is a rather poor color conductor, the color currents can still persist in the system significantly longer than the color charge density [12], as they couple to the non-perturbative chromomagnetic fields.

Since the conductivity is responsible for whitening of the quark-gluon plasma in the long-wave limit, we discuss in more detail the equation

$$\big[D^0 + \sigma\big]\rho(x) = 0 \;, \tag{35}$$

which describes how the ohmic currents neutralize the system. Eq. (35) is solved by

$$\rho(x) = \Omega(x, x_0)\, e^{-\sigma t}\rho_0(\mathbf{x})\, \Omega(x_0, x) \;, \tag{36}$$

where $x \equiv (t, \mathbf{x})$, $x_0 \equiv (0, \mathbf{x})$ and $\Omega(x, x_0)$ is the parallel transporter

$$\Omega(x, x_0) = \mathcal{T}\exp\Big[ig\int_0^t dt' A^0(t', \mathbf{x})\Big] \;, \tag{37}$$

with $\mathcal{T}$ denoting the time ordering. Observing that

$$\Big(\frac{\partial}{\partial t} - igA^0(x)\Big)\Omega(x, x_0) = \Omega(x_0, x)\Big(\frac{\overleftarrow{\partial}}{\partial t} + igA^0(x)\Big) = 0 \;, \tag{38}$$

one shows by direct calculation that the expression (36) solves Eq. (35). Since σ has non-diagonal entries, various colors are coupled to each other in the course of temporal evolution.

IV. DISCUSSION

As discussed in our previous paper [3], parton-parton collisions thermalize the momentum degrees of freedom but they do not neutralize the local color charges. To whiten the quark-gluon plasma collective phenomena are required. The local charges generate chromoelectric fields, which, in turn, induce color currents. At the scale longer than $\sqrt{\mathcal{D}/\sigma}$, these ohmic currents effectively neutralize the system, more effectively than the diffusive currents caused by the charge density gradients.

The question arises what is the characteristic time scale of momentum thermalization and that of the plasma whitening. Our analysis implicitly assumes that the equilibration of momentum is much faster than the neutralization, as we linearize the transport equations around the local equilibrium distribution functions which cancel the collision terms. In other words, it is implicitly assumed that the plasma momentum distribution first reaches its local equilibrium form, and then the system is neutralized. Unfortunately, we are unable to compute the two scales of interest as it requires an analysis of parton-parton scattering in a colorful non-equilibrium configuration. Our choice of local equilibrium configuration found in [3] is to some extent dictated by technical reasons. The local equilibrium distribution functions represent a nontrivial colorful configuration that is convenient to compute transport coefficients as the collision terms then vanish.

The problem of plasma equilibration is also complicated by the fact that color collective phenomena are not only responsible for the whitening but they also contribute to the momentum equilibration. One of us has argued for long time [21–24], see also [25,26], that color plasma instabilities, which occur in anisotropic systems, speed up the momentum thermalization. If the plasma momentum distribution is strongly elongated in one direction, as it occurs in heavy-ion collisions, the instabilities generate momentum in the transverse direction, making the system more isotropic. However, the instabilities also generate local color charges that have to be neutralized. Thus, the whole process of equilibration of the quark-gluon plasma is very complex, and it depends on the plasma initial state.

The only reliable estimates of the time scales of interest have been found for the perturbative quark-gluon plasma which is close to global equilibrium. To equilibrate the system's momentum degrees of freedom, the parton-parton interactions with momentum transfers of order T are needed. Such a transfer can be achieved in a single parton-parton collision or as a cumulative effect of many soft scatterings. The time scale of such processes is [16]

$$t_{\rm hard} \sim \frac{1}{g^4 \ln(1/g)\, T} \;. \tag{39}$$

As argued in the previous section, the whitening of the quark-gluon plasma occurs at $t \gtrsim t_{\rm soft}$. Thus, the plasma becomes white first and then the momentum degrees of freedom thermalize as $t_{\rm soft} \ll t_{\rm hard}$. Analogous analysis for a colorful background should include the effect of the colored chemical potentials that might alter the above picture.

At the end, let us recapitulate our considerations. Within the QCD transport theory we have found the conductivity and diffusion coefficients in the colorful equilibrium configuration. While the diffusion constant is proportional to the unit matrix in color space, the conductivity coefficient has a nontrivial tensorial structure. The macroscopic equation describing the temporal evolution of the color charge density has been derived. Its solution shows that the ohmic currents dominate whitening of the quark-gluon plasma at sufficiently long scale.

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APPENDIX A

In this Appendix we argue that the commutators $[D^0, \sigma]$ and $[\mathbf{D}, \sigma]$ are small in a perturbative regime, but we first show that $[D^\mu, \sigma] = (\widetilde{D}^\mu \sigma)$ where $\widetilde{D}^\mu$ is the covariant derivative of rank 2.

We are interested in the following expression

$$\mathbf{D}_{aa'}(\sigma^{a'b}\mathbf{E}_b) = \Big(\delta_{aa'}\nabla + g f^{aea'}\mathbf{A}^e\Big)\,(\sigma^{a'b}\mathbf{E}_b)\ . \tag{A1}$$

Because of the antisymmetry of the structure constants f^{abc}, Eq. (A1) can be rewritten as

$$\mathbf{D}_{aa'}(\sigma^{a'b}\mathbf{E}_b) = \mathbf{D}_{aa'}(\sigma^{a'b}\mathbf{E}_b) + g f^{beb'}\mathbf{A}^e\sigma^{ab'}\mathbf{E}_b + g f^{beb'}\mathbf{A}^e\sigma^{ab}\mathbf{E}_{b'} = \Big(\widetilde{\mathbf{D}}^{aa'}_{bb'}\sigma^{a'b'}\Big)\,\mathbf{E}_b + \sigma^{ab}\mathbf{D}_{bb'}\mathbf{E}_{b'}\ , \tag{A2}$$

where

$$(\widetilde{\mathbf{D}})^{ac}_{bd} \equiv \nabla\delta^{ac}\delta^{bd} + g f^{aec}\delta^{bd}\,\mathbf{A}^e + g f^{bed}\delta^{ac}\,\mathbf{A}^e\ . \tag{A3}$$

In matrix notation Eq. (A2) gets the form

$$\mathbf{D}(\sigma\mathbf{E}) = \sigma\,\mathbf{D}\mathbf{E} + (\widetilde{\mathbf{D}}\sigma)\,\mathbf{E}\ , \tag{A4}$$

and thus

$$[D^\mu, \sigma] = (\widetilde{D}^\mu\sigma)\ . \tag{A5}$$

We are now going to show that $[D^\mu, \sigma]$ is negligible when $1/g \gg 1$. We actually demonstrate that $(D^\mu\sigma_q)$, computed in the fundamental representation, is suppressed by powers of g when compared to $\sigma_q D^\mu$. The same analysis can be done in the adjoint representation, and for the gluons, but it requires tedious manipulations with color indices.

We first compute $\mathbf{D}\sigma_q$. Because of the local isotropy of the equilibrium state, we have

$$\mathbf{D}\sigma_q = \frac{g^2}{3\gamma}\int\frac{d^3p}{(2\pi)^3}\,\frac{1}{E_p}\,\mathbf{D}\big(Q^{\rm eq} + \bar{Q}^{\rm eq}\big) = \frac{g^2}{\gamma}\int\frac{d^3p}{(2\pi)^3}\,\frac{\mathbf{v}}{E_p}(\mathbf{v}\cdot\mathbf{D})\big(Q^{\rm eq} + \bar{Q}^{\rm eq}\big)\ , \tag{A6}$$

where we have used the fact that $\mathbf{v}^2 = 1$. Using the transport equation (7), Eq. (A6) is rewritten as

$$\mathbf{D}\sigma_q = \frac{g^2}{\gamma}\int\frac{d^3p}{(2\pi)^3}\,\frac{\mathbf{v}}{E_p}\,\Big(-D^0((Q^{\rm eq} + \bar{Q}^{\rm eq}) + \frac{g}{2}\{\mathbf{E}, \nabla_p(Q^{\rm eq} - \bar{Q}^{\rm eq})\} + (L[\delta Q] + \bar{L}[\delta\bar{Q}])\Big)\ . \tag{A7}$$

The first term in the r.h.s. of Eq. (A6) vanishes because of local isotropy. The remaining two terms are nonzero but $\mathbf{D}\sigma_q\mathbf{E}$ is seen to be smaller than $\sigma_q\mathbf{D}\mathbf{E}$ by at least two powers of g. The terms with the collision terms are even more suppressed.

Let us now discuss $D^0\sigma_q$. Using the transport equation (7), one gets

$$D^0\sigma_q = \frac{g^2}{3\gamma}\int\frac{d^3p}{(2\pi)^3}\,\frac{1}{E_p}\Big(-\mathbf{v}\nabla(Q^{\rm eq} + \bar{Q}^{\rm eq}) + \frac{g}{2}\{\mathbf{E}, \nabla_p(Q^{\rm eq} - \bar{Q}^{\rm eq})\} + (L[\delta Q] + \bar{L}[\delta\bar{Q}])\Big)\ . \tag{A8}$$

The first and the second term in the r.h.s. of Eq. (A8) both vanish because of local isotropy of the equilibrium momentum distribution. The third term does not vanish but $(D^0\sigma_q)\rho$ is suppressed with respect to $\sigma_q D^0\rho$ by powers of g hidden in L and $\bar{L}$.

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